ELECTRON HEAT CONDUCTIVITY OF THE PLASMA ACROSS A "BRAIDED" MAGNETIC FIELD

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Abstract

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Small perturbations of magnetic fields in toroidal magnetic traps can lead to a very complicated behaviour of magnetic field lines. These fields are considered to be random, and both magnetic line diffusion and electron heat conductivity produced by the diffusion of magnetic field lines are studied.

1. INTRODUCTION

Small perturbations of magnetic fields in toroidal magnetic traps can result in appreciable changes in the magnetic field line topology. 'Magnetic islands' [1] or even regions with destroyed magnetic surfaces [2–4] may appear in a plasma. Particularly, such a destruction occurs near separatrix surfaces [5,6]. A slight destruction of magnetic surfaces ('magnetic flutter' [7]) can be produced by microinstabilities.

Spontaneous or outside-induced appearance of regions with a stochastic field followed by the destruction of magnetic surfaces can affect the transport processes in the plasma. The electron thermal conductivity appears to be especially sensitive to this effect [8,9]. Here, we shall consider the effect in MHD-approximation in detail.

2. MAGNETIC-FIELD DIFFUSION

We start by considering the behaviour of a magnetic field line in the simplest case where a small transverse random field $\vec{B}'$ is superimposed on a strong homogeneous magnetic field $\vec{B}_0$. We assume that $b = B'/B_0 \ll 1$ is a space-homogeneous random function and $\langle b^2 \rangle = b_0^2$. Let the z-axis of the orthogonal co-ordinates be directed along $\vec{B}_0$. Then, starting from the origin of the co-ordinates (0,0) along the field line, we have for a transverse excursion of the field line $\vec{r}_\perp$:

$$\vec{r}_\perp = \int_0^z \vec{b}(z, \vec{r}_\perp)\, dz \tag{1}$$

where $\vec{b} = \vec{B}'/B_0$. The $\vec{r}_\perp$-vector in the integrand of expression (1) can be replaced by zero for $b \ll 1$. It is natural to call this approximation quasi-linear. On averaging we obtain for large z:

$$\langle r_\perp^2 \rangle = \int_0^z \int_0^z \langle \vec{b}(z', 0)\, \vec{b}(z'', 0) \rangle\, dz'dz'' = 4D_F z \tag{2}$$

where

$$D_F = \frac{1}{4} \int_{-\infty}^{\infty} \langle \vec{b}(z, 0)\vec{b}(0,0) \rangle\, dz \tag{3}$$

D_F is the diffusion coefficient of the field lines [8]. Expression (3) may be written in dimensional form:

$$D_F = \frac{1}{4} b_0^2\, L_0 \tag{4}$$

where L_0 is the longitudinal correlation length determined by relations (3) and (4). As is easy to see, we have in the quasi-linear approximation, at $z \gg L_0$:

$$\langle r_\perp^2 \rangle = b_0^2 L_0 z \tag{5}$$

Let δ be the transverse correlation length of the magnetic fields. From expression (1), we see that the assumption $\vec{r}_\perp = 0$ in the integrand is valid only when the field line is slightly displaced from its initial position over a length of L_0, i.e. when $\langle r_\perp^2 \rangle_{z=L_0} = b_0^2 L_0^2 \ll \delta^2$. Introducing the parameter $R = b_0 L/\delta$, we write the condition for the quasi-linear approximation to be valid in the following form:

$$R = b_0 L_0/\delta \ll 1 \tag{6}$$

To understand what happens in the opposite case when $R \gtrsim 1$ we first consider the limiting case $R = \infty$, i.e. a random field homogeneous in z. Since B_0 = const, div $\vec{b} = 0$ and, hence, we can introduce the flux function ψ:

$$\vec{b} = -[\vec{e}_z \nabla\psi] \tag{7}$$

where $\vec{e}_z$ is the unit vector along z. Now, the equations of the field line walk are reduced to Hamiltonian-type equations:

$$\frac{\partial x}{\partial z} = \frac{\partial \psi}{\partial y}; \quad \frac{\partial y}{\partial z} = -\frac{\partial \psi}{\partial x} \tag{8}$$

When ψ is independent of z, expression (8) can be considered an "equation of motion" for particles with a random stationary Hamiltonian ψ, z playing the role of time.

As is seen from expression (8), the field lines go along the lines ψ = const which can be considered to be the constant-level lines of "a topographical map", where ψ is the altitude above "water" level. If $\langle\psi\rangle = 0$, $\langle\psi^2\rangle = \psi_0^2$, the map appears to be composed of "hills" and "lakes", the averaged depth of lakes and height of hills being given by the ψ_0-value.

Let ψ be equal to a = const. If $a > \psi_0$ then only the highest separate hills will be found at this level and the corresponding lines will be singly closed loops. As the a-value decreases, the number of hills starts increasing, their dimensions increase, too, and then they start to coalesce, and at the instant of coalescence the $\psi = a$ plane passes through "the passes", i.e. through hyperbolic points. The length of field lines will increase in such a process.

A similar situation prevails when the level $\psi = a$ is chosen from the side of "the lakes", i.e. for $a < 0$.

The behaviour of the ψ = const-lines for a randum ψ function has been analysed in the problem of current percolation in random inhomogeneous solids [10,11].

It has been shown that $\psi = a$-lines are closed for all $\psi = a \neq 0$, but the mean length ℓ_ψ of such line in the (x,y) plane tends to infinity as $\ell_\psi \sim \psi^{-\gamma}$ when ψ goes to zero. (Here, γ = const $\cong 2.4$) [12]. Thus, the mean line length is $\langle\ell_\psi\rangle = \int \ell_\psi P(\psi) d\psi = \infty$ ($P(\psi)$ is the ψ distribution function). In other words, the field lines can walk any large distance on the average. The lines with small ψ which make the major contribution to the line "transport" over a large distance, wander at random, passing the hyperbolic points. Since the average "velocity" of the field line is b_0 and the average distance between the hyperbolic points is proportional to δ, the diffusion coefficient in the $R \gtrsim 1$ region can be estimated to be of the order of magnitude of $D_F \sim b_0 \delta$.

To determine D_F more precisely (but still approximately) in the whole region of variation of R we shall use the following simplified picture. Let N be constant along the field lines:

$$\frac{\partial N}{\partial z} + \vec{b} \cdot \nabla N = 0 \tag{9}$$

This value can be considered to be some "density" of labelled field lines. We split N into two components: a slowly varying N_0 and a fluctuating N': $N_0 + N'$. Now, we average expression (9) over the stochastic field $\vec{b}$, assuming $\langle N \rangle' = 0$. We would like to derive a diffusion-type equation for N_0; hence we assume that averaging Eq.(9) would yield:

$$\frac{\partial N_0}{\partial z} = -\mathrm{div}\langle \vec{b} N' \rangle = D_F \Delta_\perp N_0 \tag{10}$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$. It is natural to expect an equation of this form because of the diffusional character of the walk of lines. From Eqs (9) and (10), we obtain for N':

$$\frac{\partial N'}{\partial z} + \vec{b} \cdot \nabla N' - \langle \vec{b} \cdot \nabla N' \rangle = -\vec{b} \cdot \nabla N_0 \tag{11}$$

The equation for N' is of the same form as for N_0 but there is a right-hand side. On a scale $\geqslant \delta$, the homogeneous expression (11) for N' would again be of the form of expression (10), with the only difference that the right-hand side of expression (11) "feeds" perturbations of the δ-scale and "the diffusion coefficient" on such a scale can differ from D_F. If we neglect this difference and write approximately expression (11) in the form

$$\frac{\partial N'}{\partial z} - D_F \Delta_\perp N' = -\vec{b} \cdot \nabla N_0 \tag{12}$$

then Eqs (10) and (12) will form a closed set of equations to determine D_F. Equation (12) is easily solved by using a Fourier transformation. Substituting this solution into Eq.(10), we find

$$D_F = \frac{1}{2} \int b_{\vec{k}}^2 (ik_z + D_F k_\perp^2)^{-1} d\vec{k} \tag{13}$$

where the factor 1/2 is the result of averaging over the angles, and $b_{\vec{k}}^2$ is determined from the following expression:

$$b_{\vec{k}}^2 = (2\pi)^{-3} \int \langle \vec{b}(0) \cdot \vec{b}(\vec{r}) \rangle \, e^{-i\vec{k}\cdot\vec{r}} \tag{14}$$

For small b^2, $D_F k^2$ in the denominator of Eq.(13) is small and we obtain from (13):

$$D_F = \frac{\pi}{2} \int \delta(k_z) \, b_{\vec{k}}^2 \, d\vec{k} \tag{15}$$

This is just a quasi-linear expression coinciding with Eq.(3). On the contrary, when $D_F k^2$ is large, k_z in the denominator of Eq.(13) can be neglected and then we have

$$D_F^2 = \frac{1}{2} \int \psi_{\vec{k}}^2 \, d\vec{k} \tag{16}$$

where $\psi_{\vec{k}}$ is the Fourier component of the correlation function for ψ:

$$\psi_{\vec{k}}^2 = (2\pi)^{-3} \int \langle \psi(0) \psi(\vec{r}) \rangle e^{-i\vec{k}\cdot\vec{r}} \, d\vec{r} \tag{17}$$

According to expression (16), $D_F \sim b_0 \delta$. So, for small values of $R = b_0 L_0/\delta$ the value of D_F increases as b_0^2 with b_0 and when $b_0 L_0/\delta > 1$, the increase is only linear.

3. ELECTRON THERMAL CONDUCTIVITY IN MHD-APPROXIMATION

3.1. We now consider the behaviour of the electron thermal conductivity for the same case of a random stationary magnetic field. We assume that the longitudinal thermal conductivity $\chi_\parallel$ substantially exceeds the transverse one, $\chi_\perp$, i.e. $\Omega\tau = \sqrt{\chi_\parallel/\chi_\perp} \gg 1$ (at some stages we shall use the small parameter $\gamma = 1/\Omega\tau$). The heat flux $\vec{q}$ is equal to

$$\vec{q} = -\chi_\parallel \vec{h}(\vec{h}\nabla) - \chi_\perp T \tag{18}$$

where $\vec{h} = \vec{B}/\vec{B}_0 \cong \vec{e}_z + \vec{b}$. We see from this expression that for $b_0 \Omega\tau > 1$ the contribution of longitudinal thermal conductivity to effective heat transport across an averaged magnetic field can be substantially larger than that of transverse thermal conductivity.

We again start with a quasi-linear approximation. Let us suppose that $T = T_0 + T'$ and $T' \ll T_0$. Under stationary conditions, div $\vec{q} = 0$. The linear part of this equation in the Fourier transformation when the gradient ∇T_0 is directed along the X-axis takes the following form:

$$(k_z^2 + \gamma^2 k_\perp^2)T' = ik_z b_x \frac{dT_0}{dx} \tag{19}$$

We include, in the expression for the averaged heat flux, only the terms with the transverse conductivity:

$$\langle q_x \rangle = -\chi_\| \langle b_x^2 \rangle \frac{dT_0}{dx} - \chi_\| \langle b_x \frac{dT'}{dx} \rangle \tag{20}$$

Substituting the expression for T' from Eq.(19) we obtain the value for the proportionality coefficient between $\langle q_x \rangle$ and dT_0/dx:

$$\chi_F = \frac{1}{2} \chi_\| \int \frac{\gamma^2 k_\perp^2 b_k^2}{k_z^2 + \gamma^2 k_\perp^2} d\vec{k} \tag{21}$$

The factor 1/2 again appears, owing to an averaging over angles since $b^2 = b_x^2 + b_y^2$. As was shown before, the validity condition for the quasi-linear approximation for a magnetic field is an inequality, $b_0 L_0/\delta \sim b_0 k_\perp / k_z \ll 1$. On the other hand, it is sufficient to consider the region $b_0 \Omega\tau > 1$. Hence, in this region, $\gamma k_\perp \ll k_z$ and Eq.(21) may be written as follows:

$$\chi_F = \frac{\pi}{2} \sqrt{\chi_\| \chi_\perp} \int k_\perp \delta(k_z) b_k^2 d\vec{k} \tag{22}$$

Comparing with expression (15), we see that $\chi_F = \sqrt{\chi_\| \chi_\perp} \langle k_\perp \rangle D_F$, where $\langle k_\perp \rangle$ is the averaged transverse wavenumber. By order of magnitude, $\chi_F \sim \sqrt{\chi_\| \chi_\perp}\, b_0^2 L_0 \delta^{-1}$. Let us recall that the Bohm diffusion coefficient is equal to $D_B = (1/16)\sqrt{\chi_\| \chi_\perp}$, so that the dependence of χ_F on the plasma parameters is a Bohm-type one. For $b_0 \sim \delta/L_0$, we have $\chi_F \sim 16\, b_0 D_B$.

Note that the expression for χ_F can be written in the form: $\chi_F = \bar{v} D_F$, where $\bar{v} = \chi_\|/L$, $L \sim \delta\Omega\tau$. The quantity $\bar{v}$ plays the role of velocity of heat transport over a characteristic length L, i.e. the longitudinal correlation length for T'.

3.2. According to expression (22), χ_F tends to zero as $\chi_\perp \to 0$, but this result does not seem to be reliable because we neglect the terms of higher order in b in Eq.(19). Let us consider, instead of Eq.(19), a more accurate equation with a "point" source with respect to Z:

$$\left(\frac{\partial}{\partial z} + \vec{b}\cdot\nabla\right)^2 T + \gamma^2 \nabla_\perp^2 T = \delta(z) e^{i\vec{k}_\perp \cdot \vec{r}_\perp} \tag{23}$$

Neglecting the term with $\vec{b}\cdot\nabla$ we could obtain an equation for the function of the point source. It is this function which was used in Eq.(19). The presence of the term $\vec{b}\cdot\nabla$ implies that the point source function, while remaining a comparatively slow-varying function of the variable ℓ along the field line, oscillates strongly in $\vec{r}_\perp$, owing to "braiding" of the field lines. The result of this is the increase of the term $\nabla_\perp^2 T$ (see, e.g. Ref.[9]). We should include this effect.

Note that Eq.(23) can be derived from the variation principle $\delta S = 0$, where

$$S = \int \left\{\left(\frac{\partial T}{\partial \ell}\right)^2 + \gamma^2 (\nabla_\perp T)^2\right\} d\vec{r} \tag{24}$$

Here, $\partial/\partial\ell = \partial/\partial z + \vec{b}\cdot\nabla$. We average expression (24) over $\vec{b}$ and approximate this by the following equation:

$$S' = \langle S \rangle = \int \left\{\left(\frac{\partial T}{\partial z}\right)^2 + \Gamma^2(z) T^2\right\} d\vec{r} \tag{25}$$

where $\Gamma(z)$ remains to be found. To do this, we should find $\langle(\nabla_\perp T)^2\rangle$ as a function of z. Let us write $\partial T/\partial x = (T(x_2) - T(x_1)/(x_2 - x_1)_{x_2 - x_1 \to 0}$. Here $T(x,\ell)$ is a slowly varying function of ℓ (since the longitudinal conductivity is high), so that the expression for $\partial T/\partial x$ can approximately be written as follows: $\partial T/\partial x = (\partial T/\partial x)_0 \cdot (x_2^0 - x_1^0)/(x_2 - x_1)$, where x_i^0 are co-ordinates of trajectories at Z = 0. When the difference $x_2 - x_1$ along the trajectory is small,

$$\frac{\partial}{\partial x}(x_2 - x_1) = b_x(x_2, z) - b(x_1, z) = \frac{\partial b_x}{\partial x_z}(x_2 - x_1) \tag{26}$$

From this, we find

$$x_2 - x_1 = (x_2^0 - x_1^0) \exp\left(\int_0^z \frac{\partial b_x}{\partial x} dz\right)$$

Assuming a Gaussian distribution for $\vec{b}$ we find, for large values of z, for the T function, which is proportional to $\exp(i\vec{k}_\perp \cdot \vec{r}_\perp$ at z = 0, the following expression:

$$\left\langle \left(\frac{\partial T}{\partial x} \right)^2 \right\rangle = k_x^2 T^2 \exp(2\,kz)$$

where

$$k = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\partial b_x(0)}{\partial x} \frac{\partial b_x(z)}{\partial x} \, dz \tag{27}$$

By order of magnitude, $k \sim b_0^2 L_0 \delta^{-2}$. In the region $z < 0$, we have to substitute $\exp(2k|z)$. But since this exponent is essential only for large values of z we shall cover both positive and negative z-values by approximating the two exponents by the function ch2kz. Thus,

$$S = \int \left\{ \left(\frac{\partial T}{\partial z} \right)^2 + \gamma^2 k_\perp T^2 \,\mathrm{ch}(2kz) \right\} dz \tag{28}$$

Varying this functional we obtain a modified Mathieu equation. An approximate value of T can, however, be found directly from expression (28). For small k, it naturally yields the former quasi-linear expression. For very small γ, we find the solution for T in the form $1-\alpha Z$ at $Z < \alpha^{-1}$, $T = 0$ at $Z > \alpha^{-1}$ (α is the variable parameter). This is due to the exponentially strong "switching-on" of the transverse damping in expression (28). The variation of expression (28) with such a test function gives:

$$\alpha \cong k(\ln k/\gamma k_\perp)^{-1} \tag{29}$$

By magnitude, $\alpha \sim k \sim b_0^2 L_0 \delta^{-2}$. As we see, $k_z \sim k\alpha^{-1} \gg 1$ only when the logarithm in expression (29) is large, i.e. when

$$\Omega \tau b_0 \gg (b_0 L_0/\delta)^{-1} \tag{30}$$

If we approximate the Green function for Eq.(23) $(1/2\alpha)(1-\alpha|z|)$ by a simpler expression $(1/2\alpha)\exp(-\alpha|z|)$, then after substitution of the corresponding solution for T′ into Eq.(20) we obtain an expression of the form (21) but with a term α^2 instead of $\gamma^2 k_\perp^2$. In the quasi-linear region $b_0 L_0 \ll \delta$ we can again assume

$\alpha^2 \ll k_z^2$ (k_z is the characteristic wave number), so that, instead of expression (22), we obtain

$$\chi_F = \chi_\parallel \alpha D_F \sim \chi_\parallel b_0^4 L_0^2 \delta^{-2} \quad (31)$$

This expression is valid in the region $b_0 L_0/\delta \ll 1$, $\Omega \tau b_0 \gg (b_0 L_0/\delta)^{-1}$.

3.3. Let us now consider the region $b_0 L_0/\delta > 1$, where a quasi-linear approximation is not valid. We again start with a simpler case where $L_0 = \infty$, i.e. a case of a random field homogeneous in Z.

The problem in question is similar to that of current percolation in a two-dimensional solid with random inhomogenous conductivity [11]. So, it seems reasonable to use a well-known method by Dykhne [14], which allowed him to obtain, in a rather simple way, an expression for the effective electrical conductivity of an inhomogeneous substance. Let us write Eq.(18) in the following form:

$$q_\alpha = -\chi_{\alpha\beta} \frac{\partial T}{\partial x_\beta} \quad (32)$$

where $\chi_{\alpha\beta} = b_\alpha b_\beta \chi_\parallel + \delta_{\alpha\beta}\chi_\perp$, and the sum is taken over subscript β. When the averaged gradient is directed along the x-axis, Eq.(32), on averaging, takes the form:

$$\langle q_z \rangle = -\chi_F \frac{dT_0}{dx} \quad (33)$$

where χ_F is the value of the effective temperature conductivity to be found. Now, we rotate the whole picture of gradient and heat fluxes through an angle of 90° and introduce the following quantities:

$$\vec{q}' = A[\vec{e}_z \times \nabla T]; \quad \nabla T' = B[\vec{e}_z \times \vec{q}\,] \quad (34)$$

where A and B are constants. Since div $\vec{q} = 0$, rot $\nabla T = 0$, we have div $\vec{q}\,' = 0$, rot $\nabla T' = 0$, i.e. the $\vec{q}\,'$ and $\nabla T'$ values can be considered as heat flux and temperature gradient in the turned plane. Now we try to choose values of A and B such that the turned picture is as close to the initial one as possible.

The linear relation between $\vec{q}$ and ∇T gives:

$$q'_\alpha = -\chi'_{\alpha\beta} \frac{\partial T'}{\partial x_\beta} \quad (35)$$

and an averaged heat flux is equal to

$$\langle q'_x \rangle = -\chi'_F \frac{dT'_0}{dx} \tag{36}$$

Now we choose A and B such that, on the average, the heat transports in the initial and turned planes coincide or at least, are as close to one another as possible. For this purpose, the following requirements should be met:

$$\chi_F = \chi'_F, \qquad \langle \chi_{\alpha\beta} \rangle = \langle \chi'_{\alpha\beta} \rangle \tag{37}$$

Averaging expressions (34) and making use of Eqs (33), (36) and the first equation (37), we find

$$\chi_F = \sqrt{A/B} \tag{38}$$

On the other hand, A/B can be found from the second equation (37). Taking into account that, according to Eqs (34) and (35), $\chi'_{\alpha\beta} = A/B\, \chi^{-1}_{\alpha\beta}$ (where $\chi^{-1}_{\alpha\beta}$ is the inverse matrix) we can find from the second equation (37):

$$A/B = \langle \chi_{xx} \rangle \left\langle \frac{\chi_{xx}}{\chi_{xx}\chi_{yy} - \chi^2_{xy}} \right\rangle^{-1} = \chi_\parallel \chi_\perp \langle b^2_x \rangle \left\langle \frac{b^2_x}{b^2} \right\rangle^{-1} = \chi_\perp \chi_\parallel b^2_0 \tag{39}$$

Here, the explicit expression for the $\chi_{\alpha\beta}$ tensor is included, and we have restricted ourselves to the case $b_0 \Omega\tau \gg 1$. Thus, according to expressions (38) and (39), we have

$$\chi_F = \sqrt{\chi_\parallel \chi_\perp}\; b_0 \tag{40}$$

in the region $b_0 L_0/\delta \gg 1$.

3.4. Formula (40) again gives $\chi_F \to 0$ as $\chi_\perp \to 0$ which may be invalid when $L_0 \neq 0$. So the region $b_0 \Omega\tau \gg 1$ should be considered in more detail for large, but finite values of the parameter $R = b_0 L_0/\delta$. To consider this more general case we first try to obtain expression (40) by using a simpler approach of the quasi-linear type. To do this, we write the expression for the heat flux (18) for a purely two-dimensional case in the following form:

$$\vec{q} = -\chi_\parallel \vec{b}\; b\frac{\partial T}{\partial s} - \chi_\perp \nabla T \tag{41}$$

where $b(\partial/\partial s) = \vec{b} \cdot \nabla$, i.e. s is the co-ordinate along the field line projection on the (x,y)-plane. If we again suppose $T = T_0(x) + T'(x,y)$ and average expression (41), we obtain, neglecting the second term:

$$\langle q_x \rangle = -\chi_\| \langle b_x^2 \rangle \frac{dT_0}{dx} - \chi_\| \left\langle bb_x \frac{\partial T'}{\partial s} \right\rangle \tag{42}$$

The value of T' can be found from the linearized equation $\operatorname{div} \vec{q} = 0$. Since $\operatorname{div} \vec{b} = 0$ this equation will read:

$$b^2 \frac{\partial^2 T'}{\partial s^2} + \gamma^2 \Delta_\perp T' = -b \frac{\partial}{\partial s} \left(b_x \frac{dT_0}{dx} \right) \tag{43}$$

When $\chi_\| \gg \chi_\perp$, the major contribution to the thermal conductivity comes from those field lines that wander through long distances (small "islands" and "lakes" practically do not contribute to the thermal conductivity). Along those lines that correspond to small ψ and can be considered unclosed, the value of s virtually varies from $-\infty$ to $+\infty$. Thus, T' can be expanded over s in a Fourier integral. As for $\Delta_\perp$, this can be assumed to be equal to $-k_\perp^2$ for a perturbation of the typical form $\exp(i\vec{k}_\perp \cdot \vec{r}_\perp)$. Thus, from Eqs (42) and (43), we obtain an expression similar to (21), with the only difference that here we have the term $b^2 k_s^2$ instead of k_z^2:

$$\chi_F = \frac{1}{2} \chi_\| \int \gamma^2 k_\perp^2 b_{\vec{k}}^2 (b_{k_s^2}^2 + \gamma^2 k_\perp^2)^{-1} \, d\vec{k} \tag{44}$$

where k_s is the wave number along s. For small γ in expression (44) we again see the occurrence of a δ-function; thus, the expression will read:

$$\chi_F = \sqrt{\chi_\| \chi_\perp} \, \frac{\pi}{2} \int k_\perp \delta(bk_s) b_{\vec{k}}^2 \, d\vec{k} \tag{45}$$

Since $b_{\vec{k}}^2$ differs from zero in the region $k_s \sim k_\perp \sim \delta^{-1}$, expression (45) yields the same estimate as (40), i.e. $\chi_F \sim \sqrt{\chi_\| \chi_\perp} \, b_0$.

For $L_0 \neq \infty$, the heat can flow along the field lines, and this should contribute to $\langle q_x \rangle$. In the region $b_0 L_0/\delta \gg 1$, the expression for the averaged flux can be taken in the form (42), the small gradient along z being neglected. Thus, it is sufficient to take into account additional damping due to $\partial/\partial z$ only in Eq.(43). To do this, we again make use of the functional (24) and first average it over $\vec{b}$:

$$S' = \langle S \rangle = \int \left\langle b_0^2 \left(\frac{\partial T'}{\partial s} \right)^2 + \left(\frac{\partial T'}{\partial z} \right)^2 + \gamma^2 (\nabla_\perp T')^2 \right\rangle d\vec{r} \tag{46}$$

We have again introduced here the s-co-ordinate along the field line projection onto the (x,y)-plane and have "split" approximately the average squares of the functions in the first term in the integrand. Expression (48) can be considered to be a functional whose variation gives the point source function in the s-variable.

In expression (46), the second and third terms in the integrand are small and so T' in these terms can be considered almost constant in s, i.e. approximately $T' \cong T'(\psi)$. In this case, the average value of the second and third terms will be expressed in terms of correlation properties of ψ, i.e. expression (46) can be written as follows:

$$S' = \int \left\{ b_0^2 \left(\frac{\partial T'}{\partial s}\right)^2 + \left(\frac{a_1}{L_0^2} + \gamma^2 \frac{a_2}{\delta^2}\right) T'^2 \right\} d\vec{r} \tag{47}$$

where a_1, a_2 are numerical coefficients of the order of unity. When $\Omega\tau > L_0/\delta$, the term with γ^2 can be neglected and the damping is determined by the value of a_1/L_0. If we now take a quasi-linear-type equation (44) and replace $\gamma^2 k_\perp^2$ by a_1/L_0^2 we approximately obtain:

$$\chi_F \cong \chi_\parallel b_0 \delta L_0^{-1} \tag{48}$$

This expression is valid for the region $b_0 L_0/\delta > 1$, $b_0 \Omega\tau > b_0 L_0/\delta$.

3.5. Figure 1 summarizes the results of our considerations. In this figure four regions are presented for χ_F as a function of the parameters $b_0\Omega\tau$ and $b_0 L_0/\delta$. $b_0\Omega\tau$ lies in the $b_0\Omega\tau > 1$ region and the value of $b_0 L_0/\delta$ may be arbitrary. In all four regions, $\chi_F \cong \bar{v} D_F$, where D_F is the field diffusion coefficient and $\bar{v} = \chi_\parallel/L$, where L is the characteristic length over which the temperature perturbation is transported. In regions 1 and 4, the value of $L \sim \delta\Omega\tau < L_0$, in region 3 $L \sim L_0$, in region 2 $L \sim L_0 (b_0 L_0/\delta)^{-2} > L_0$. On the boundaries of the regions different dependences match.

The expression $\chi_F = \bar{v} D_F$ for $\bar{v} = \chi_\parallel/L$ is valid only at $\bar{v} < v_{Te}$, where v_{Te} is the thermal electron velocity, i.e. in a hydrodynamic collisional regime, $\lambda_e < L$. For $\lambda_e > L$, as has been shown by Stix [8] and Rechester and Rosenbluth [9], $\chi_F \cong v_{Te} D_F$.

4. OHKAWA FORMULA

In a high-temperature, high-electric-conductivity plasma, it is natural to expect a longitudinal field component B'_x to arise because of plasma displace-

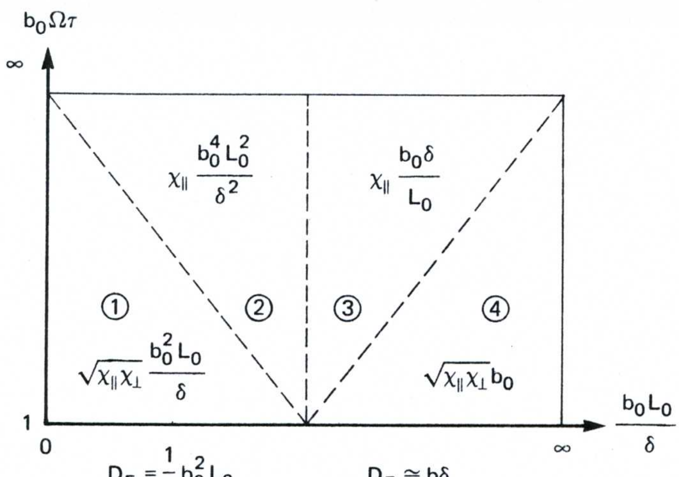

FIG.1. Four regions displaying various dependences of heat conductivity in a "braided" magnetic field.

ments. It is also natural to assume that the displacement is of the order of the longitudinal correlation length, so that $B' \sim B_0\delta/L_0$. In other words, the most natural value of the parameter b_0L_0/δ is unity. According to Fig.1, at $b_0L_0/\delta \sim 1$, χ_F is virtually independent of $\Omega\tau$ and equal to $\chi_F \sim \chi_\parallel(\delta^2 L_0^2)$ in a collisional regime, and to $\chi_F \sim v_{Te}\delta^2 L^{-1}$ in a collisionless regime. In tokamak-type systems, it is reasonable to assume that $L_0 = qR$, where R is the major torus radius, and q the safety factor. Hence, in the plateau (and banana) regimes, $\lambda_e > qR$, and we should use the expression $\chi_F \sim v_{Te}\delta^2/qR$.

To describe the anomalous electron thermal conductivity, Ohkawa has proposed the formula:

$$\chi_0 \cong \frac{c^2}{\omega_{pl}^2}\,\frac{v_{Te}}{qR} \tag{49}$$

which correspond to the assumption $\delta \cong c/\omega_{pl}$ (ω_{pl} is the Langmuir frequency). This formula is in good agreement with the experimental data.

Evidently, the Ohkawa formula corresponds to the assumption that magnetic surfaces are destroyed on the $\delta \sim c/\omega_{pl}$ scale. This assumption may, to some extent, correspond to the hypothesis of a magnetic flutter [7]. In fact, if we take into account the fact that in drift oscillations the magnetic surfaces are frozen into the electrons, then the destruction of surfaces and reconnection of field lines in a tenuous plasma can be expected only on the scale of a collisionless skin layer,

$\delta \sim c/\omega_{pl}$. That is why formula (49) seems to be quite reasonable in explaining the anomalous electron heat conductivity under the conditions of magnetic surface "flutter" in tokamaks.

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DISCUSSION

S. INOUE: If diffusion is governed by the parallel motion of electrons, then the confinement of runaway electrons must be worse than that of ordinary electrons. Is this consistent with experimental findings?

B.B. KADOMTSEV: The behaviour of runaway electrons has been discussed by M. Rosenbluth. The magnetic drift of such electrons helps to prevent leakage along the field lines.

J.D. CALLEN: Could you please discuss in somewhat more detail why you have chosen c/ω_{pe} as the transverse correlation length? In particular, could you indicate why you have not taken a larger scale length such as might arise from drift waves which can also, presumably, cause fluttering of the magnetic field lines?

B.B. KADOMTSEV: If electron collisions are very rare, the drift-type waves will conserve magnetic surfaces, so that for a real reconnection of magnetic field lines a smaller scale may be important – for instance c/ω_{pe}, the collisionless skin-length.

B. COPPI: The numerical simulation of discharges obtained in Alcator and other devices has indicated that a combination of two diffusion coefficients is needed to reproduce the observed temperature profiles. One of these two coefficients controls the electron energy transport near the centre of the plasma column and has a form similar to the one you discussed, except that the exponent of the electron temperature has the opposite sign. The other gives an enhanced diffusion at the edge of the plasma column and has different dependences on n, q and T. These conclusions have been confirmed recently by three other groups who have carried out analyses similar to ours.

T. OHKAWA: The same model can be applied to calculations of particle transport as well as heat transport, the resultant flow of particles being parallel to the magnetic field lines. Perhaps this might explain the lack of experimental observation of the bootstrap effect despite the fact that the transport rate is much greater than the neoclassical rate. Would you care to comment?

B.B. KADOMTSEV: The problem of particle transport is more complicated than heat transport, owing to electric field perturbations. We have not considered this problem as yet.

M. DUBOIS: Do you explain this turbulence by fluttering of the lines of force? I would not have thought that this was likely, strictly speaking, to be a mechanism for turbulence.

B.B. KADOMTSEV: Yes; it may not be true turbulence but simply small distortions of the magnetic surfaces.

M. DUBOIS: In the case of strong turbulence in the presence of a current gradient, it seems to me that the plasma would tend to shield itself, thus reducing the stochasticity. What do you think of this possibility?

B.B. KADOMTSEV: There may be some effect of self-curing or self-restoring of the magnetic surfaces.

F.R. SCHWIRZKE: If the thermal electron conductivity is determined by stochastic magnetic field "flutter" and related temperature fluctuations, do these processes also influence the electrical conductivity?

B.B. KADOMTSEV: No, electrical conductivity is virtually unaffected by magnetic-field fluctuations.